

## INTEGRATION OF PRODUCTS AND QUOTIENTS USING SUBSTITUTION

Examples:

1)  $\int x\sqrt{x-3} \, dx$

$$u = \sqrt{x-3} = (x-3)^{1/2}$$

$$\frac{du}{dx} = \frac{1}{2}(x-3)^{-1/2} \cdot 1 = \frac{1}{2\sqrt{x-3}}$$

$$dx = 2\sqrt{x-3} \, du = 2u \, du$$

$$\text{Since } u = \sqrt{x-3} \rightarrow u^2 = x-3$$

$$\text{Then } x = u^2 + 3$$

$$\int x\sqrt{x-3} \, dx = \int (u^2 + 3) \cdot u \cdot 2u \, du$$

$$= \int 2u^4 + 6u^2 \, du$$

$$= \frac{2}{5}u^5 + \frac{6}{3}u^3 + c$$

$$= \frac{2}{5}(\sqrt{x-3})^5 + 2(\sqrt{x-3})^3 + c$$

2)  $\int \frac{2x}{(x+3)^2} \, dx$

$$u = x + 3 \rightarrow x = u - 3$$

$$\frac{du}{dx} = 1 \rightarrow dx = du$$

$$\int \frac{2x}{(x+3)^2} \, dx = \int \frac{2(u-3)}{u^2} \, du = \int \frac{2u-6}{u^2} \, du = \int \frac{2u}{u^2} - \frac{6}{u^2} \, du$$

$$= \int \frac{2}{u} - 6u^{-2} \, du = 2 \ln|u| - 6 \frac{u^{-1}}{-1} + c$$

$$= 2 \ln|x+3| + \frac{6}{x+3} + c$$

3)  $\int \frac{e^x}{3+e^x} \, dx$

$$u = 3 + e^x$$

$$\frac{du}{dx} = e^x \rightarrow dx = \frac{1}{e^x} du$$

$$\int \frac{e^x}{3+e^x} \, dx = \int \frac{e^x}{u} \cdot \frac{1}{e^x} \, du = \int \frac{1}{u} \, du = \ln|u| + c$$

$$= \ln|3 + e^x| + c$$

4)  $\int \frac{e^{\ln(x)}}{x} \, dx$

$$u = \ln(x)$$

$$\frac{du}{dx} = \frac{1}{x} \rightarrow dx = x \, du$$

$$\int \frac{e^{\ln(x)}}{x} \, dx = \int \frac{e^u}{x} \cdot x \, du = \int e^u \, du = e^u + c$$
$$= e^{\ln(x)} + c$$